CHAPTER 13

COLD ATOMS AND PRECISION MEASUREMENTS

Wencui Peng*,†,‡, Biao Tang*†,‡, Wei Yang*,†,‡, Lin Zhou*,†, Jin Wang*,†,§ and Mingsheng Zhan*,†,¶

*State Key Laboratory of Magnetic Resonance and Atomic and Molecular Physics, Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences, Wuhan 430071, China
†Center for Cold Atom Physics, Chinese Academy of Sciences, Wuhan 430071, China
‡University of Chinese Academy of Sciences, Beijing 100049, China
§wangjin@wipm.ac.cn
¶mszhan@wipm.ac.cn

1. General Introduction

Laser cooled atoms are ideal samples for fundamental science research and technology application. Cold atom physics has become a very important area of atomic and molecular physics, this area is attracting more and more attention. Cold and ultracold atoms have been widely used in experimental investigation of light-atom interaction, atom frequency standards, atom
interferometry, fundamental physical constants measurements, fundamental physics principle tests, and other precision measurements. Recently, there is great progress in cold atoms and precision measurements. Techniques of cold atom preparation are improved rapidly, and some new species of atom are cooled and trapped. Optical dipole traps for single atoms step forward to quantum information processing; ion traps for few ions provide new candidates for optical frequency standard. Implementation of cold atom interferometry has made sustained progress, new results of gravity measurement and rotation measurement based on atom interferometry are reported, and new prospects of gravitational wave detection using atom interferometers are proposed. As the most precise measurement tool of time and frequency, better stabilities are demonstrated in cold atom clocks including microwave clock, space clock, optical clock and chip clock. New determination of fundamental constants using atom interferometers, such as Newton gravitational constant and fine structure constant, are comparable with that using the classic methods. There are also some better trends in cold atom based fundamental principle tests, some projects which will offer opportunities to explore the foundations of modern physics in ground as well as in space are proposed.

In this chapter, we review the recent advances of cold atoms and their applications in precision measurements. New progresses in cold atoms preparation, cold atom interferometer, atom gravimeter, atom gravity gradiometer, atom gyroscope, gravitational wave detection, cold atom clock, determination of the gravitational constant and fine structure constant, proposal of test equivalence principle and local Lorentz invariance are reviewed in detail.

2. Progress of Cold Atoms Preparation

There are several different methods to prepare cold atoms. Magneto-optical trap (MOT) is a most popular tool for cooling and trapping neutral atoms, optical dipole trap is usually used to prepare single atoms, and ion trap is a traditional setup for confinement of few ions. There are great progresses in cold atom preparation, some new techniques of preparation of cold atoms are invented, and some new species of atoms for possible new applications are cooled and trapped.
2.1. Magneto-optical trap

MOT plays an irreplaceable role in cold atom physics. Since the first MOT was realized by Raab et al. in 1987, techniques of laser cooling and trapping of neutral atoms developed rapidly. Physicists have been trying to improve and optimize the performance of the MOTs for many years. There are two main purposes in optimizing of MOTs, the first one is to trap as many atoms as possible, and another is to cool atoms as colder as possible. Choice of atom species is quite important for better performance of a MOT and for special applications. According to different resonance wavelength and saturated vapor pressure, some species of atoms, such as rubidium, can be directly captured from the background vapor; while others, especially lighter atoms, such as lithium, need to be heated up to form atomic beam, decelerated by red detuning laser beam, and then trapped in MOT. Recently, a double MOT with Zeeman slower for $^7$Li was realized, atoms with velocities less than 100 m/s was pre-cooled from thermal lithium beam, and the capture rate of MOT was $7 \times 10^9$ atoms per second. Nowadays, most of alkali metal, alkaline earth metal and noble gas atoms can be successfully trapped in MOT. Other elements, which are used to be difficult to cool, were cooled and trapped recently, Sukachev et al. realized thulium MOT and Youn et al. realized dysprosium MOT in 2010. Both of thulium and dysprosium atoms are suitable for study of dipole–dipole interactions. The energy levels of thulium involved in cooling cycle are not perfectly closed, so the number of atoms trapped in MOT by Sukachev et al. is only $7 \times 10^4$. The level structures of lanthanides atoms are more complex, an open transition was used to cool dysprosium atoms, and a complex optical system was used to compensate atoms loss which was caused by metastable states, no more than $10^6$ atoms were trapped in Youn’s work.

To optimize MOT, it is necessary to study the factors which could affect the performance of MOT, and it is also necessary to invent new evaluation methods for the key characters of MOT. Jhe’s group added an external periodic perturbation on the laser intensity; they found that the transverse laser effect can be used to explain the discrepancies between the measured and theoretical calculated trap parameters. Van Dongen et al. determined the depth of an atom trap by the collisions of residual gas. Liebisch et al. used stimulated emission during MOT loading, atoms emit photons in a given direction due to stimulated emission, the number of trapped atoms in MOT
is 13 times more than before and the loading rate of MOT increased by nearly 20 times. Mckay et al. used a new transition to realize low temperature and high density potassium MOT, a 405 nm laser was used to drive the open transition, $^4S_{1/2} \rightarrow ^5P_{3/2}$. Compared to the previous MOT with closed transition, the temperature was several folds lower and the density was 20 folds higher.

Dual-species MOTs are good candidates for experimental investigation of interaction between different atoms and also candidates for realization of non-alkali-metal Bose–Einstein condensation (BEC). Dual-species alkali metal MOTs, such as Rb–Cs, Na–K, Li–Cs, Na–Cs, Na–Rb, Rb–Cs, K–Rb, 85Rb–87Rb, Cs–137Cs (Ref. 21) and He–Rb, (Ref. 22) were realized recently, the trap loss of loading process are investigated. Dual-Fermions MOTs provide good conditions for the study of s-wave interaction, $^3$g-factor ratios between these two Fermions, and width of Feshbach resonance. In 2011, Ridinger et al. demonstrated the realization of 6Li–40K MOT, they used a Zeeman slower to provide 6Li atom beam and a two-dimensional MOT to provide 40K atom beam, the capture rate of this dual MOT reached $10^9$ atoms per second. This dual-MOT is very useful to further study Fermions mixture.

### 2.2. Single atom trap

Single free neutral atom is one of the most promising candidates for storing and processing quantum information. Although single atom can be captured in MOT, but the MOT potential is dissipative which leads serious decoherence of single atom. For this reason, ordinary MOT is better for trapping atom cloud with millions of atoms, but it is not suitable for trapping and manipulating single atom. Optical dipole trap is a better choice for single atom traps. The first single atom trap was demonstrated by Meschede’s group in 2000; they combined the red detuned lasers and quadrupole magnetic field, increased the gradient of quadrupole magnetic field, reduced the loading rate of MOT and finally captured single atom. Hill and McClelland produced single chromium atom using a different way in 2003, they pushed an atom beam pulse to the MOT area until one atom was trapped and detected.

The ability of dipole trap to capture single atom depend on the interaction between the induced atomic electric dipole moment and the
far off resonance red detuned laser. The dipole trap can capture single atom both in free space and optical cavity. MOTs trapping atom mainly depend on radiation pressure, the dipole traps mainly depend on dipole force. Compared to MOTs, dipole traps can capture single atom more efficiently. Physicists have been changing the potential field induced by laser to capture single atom. Recently single atoms are trapped in a blue detuned optical bottle beam trap and a blue detuned hollow-beam funnel. Benefits from application of the spatial light modulator, the trap laser can be modulated as any designed patterns. In addition, to study the characteristics of single atoms, one can capture two atoms at the same time to study the two-body problems and trap several atoms in a ring lattice to study the few-body problems. Besides the application prospects in quantum information and quantum simulation, due to the lack of the interaction between atoms, the single atom trap provides a pure environment for experimental study of atomic physics.

2.3. Ion trap

The interaction between ions and external environment is stronger, and the depth of MOTs and dipole traps for neutral atoms is not deep enough to capture ions. So ions are usually trapped by special electronic potential. There are two basic types of ion traps, one is Penning trap and the other is Paul trap. Ion trap not only provide the ion source, but also can be used as a mass spectrometer. Allcock et al. demonstrated implementation of a new Paul trap in 2010, they designed a symmetric surface-electrode and applied two-photon process in trap. Zhang et al. studied the effects of varying higher-order multi-pole components, the magnitude and the scan speed can influence the mass resolution, and the mass resolution can reach a maximum value under certain condition. Benefits from the technology of ion trap, Yb\(^+\), Hg\(^+\), Ca\(^+\) (Ref. 41) and Cr\(^+\) (Ref. 42) are used in ion frequency standards.

3. Progress of Atom Interferometry

The de Broglie wavelength of microscopic atom is usually too short to show wave properties, but for cold and ultracold atoms, their wavelengths will be longer enough to display interference. Investigation of atom
interferometers and their application have made great achievements due to techniques of laser cooling, trapping and manipulation of neutral atoms. Similar to light wave interferometers, atom interferometers need beam splitting, reflecting and recombining process; these processes can be conducted either by microstructure or by standing wave of light. The principle of a typical Mach–Zehnder atom interferometer is shown in Fig. 1. Atoms are trapped in a MOT and are prepared in initial state $|a\rangle$, then the first $\pi/2$ pulse (beam splitter) is applied to atoms, and the atoms are split into Path A and Path B. Atom state in Path A keep unvaried, while atoms in Path B obtain two-photon recoil momentum, and change to state $|b\rangle$. When the $\pi$ pulse Raman beams interact with atoms in state $|a\rangle$ and $|b\rangle$, atoms obtain two-photon recoil momentum and change their states from $|a\rangle$ and $|b\rangle$ to $|b\rangle$ and $|a\rangle$, respectively. After the second $\pi/2$ pulse is applied to atoms, atom in state $|a\rangle$ and $|b\rangle$ are split again, and the new states $|a\rangle$ and $|b\rangle$ overlap spatially. In other words, initial state of atoms is entangled with the path, and detection of population in one of the ground states reveals the interference fringes. Any effect that affect on atoms will cause the phase shift of the interference fringe. If we consider the gravity field, the atoms’ path will be distorted as dashed line in Fig. 1 due to the free fall of atoms.

The first thermal atom interferometer was realized by Chebotayev et al. in 1985. (Ref. 44). They used nanostructure gratings to split atom beam. The first cold atom interferometer was realized by Kasevich and Chu in 1991 (Ref.45), they used standing wave of laser to split and recombine the atom beams. Different from nanostructure grating, stimulated off-resonance Raman transitions were used as standing wave gratings. After that, other techniques using laser field, such as Bragg diffraction, Bloch oscillations were also used to manipulate cold atoms. Laser field can also be used to choose internal or external states for atom interference; on the other hand, standing wave gratings are easier to be established than artificial nanostructure gratings.

Standing wave based cold atom interferometers are easier to realize and thus are widely used in precision measurements and fundamental physics. Kasevich and Chu considered in 1991 that the atom interferometer can be used to measure gravitational acceleration.45 In 1994, Borde et al. speculated that atom interferometers can be used to detect gravitational waves;46 Audretsch and Marzlin believed that the effect induced by space-time
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Fig. 1. Schematic diagram of a Mach–Zehnder type atom interferometer. \( \pi/2 \) pulse standing waves act as beam splitters, \( \pi \) pulse acts as reflect mirror. Solid straight lines show the idea interference path of atomic wave packet, dashed curves are interference path in gravity field.\(^4\)

Curvature can be experimentally measured with Ramsey atom interferometer.\(^4\) Chu’s group demonstrated that atom interferometer can be used to determine the fine structure constant.\(^4\) In 1997, Kasevich’s group realized an atom interferometer based gyroscope,\(^4\) they also measured the gravity gradient using two atom interferometers in 1998,\(^5\) two gravimeters shared the same lasers and reflector, and the common noises during the measurements were greatly reduced. This configuration was proposed to determine the gravitational constant with similar method. In 1999, Chu’s group\(^5\) determined the value of gravitational acceleration by atom interferometer, they analyzed the system errors, compared the experimental data of the atom interferometer gravimeter with a Michelson optical interferometer gravimeter at the same site. They found that, within the experimental error, the macroscopic glass cube in Michelson interferometer and the microscopic cesium atom fall at the same acceleration. That was the first test of weak equivalence principle (WEP) using macroscopic objects and microscopic particles.

In cold atom interferometers, the interference effect of single atom is collectively displayed. Therefore, as many as possible atoms are expected to
participate in the interference process to promote signal to noise ratio (SNR); any divergence of atom state will influence the measurement results. Atoms are also expected to be cooled as colder as possible. But, no matter how cold and how many atoms are, the interference is single-particle interference. In 2002, Gupta et al. used BECs in atom interferometer to determine the fine structure constant, the application of BECs in atom interferometer opened a new way for precision measurement. In 2004, Fermions were used in atom interferometer to measure gravity, they thought the Fermions will have better prospect in single-particle-effect type measurement due to the advantage of non-interaction.

In 2002, a project named as hyper-precision cold-atom interferometry in space (HYPER) was proposed in Europe. According to HYPER, an atom interferometer will be used to detect the Lense-Thirring effect in space. That is the second space project on cold atom and precision measurement, and the first one is on space atomic clock.

In 2004, Weitz’s group dropped the two isotope atoms $^{85}$Rb and $^{87}$Rb, and then measured the difference of their gravitational acceleration. They did not find any difference at the level of $10^{-7}$. Although the precision of testing WEP in this work is two orders of magnitude lower than the work of Chu group’s in 1999, they used two kind of microscopic elementary particles as the test masses, which had special scientific significance for WEP test.

In 2006, Weel et al. analyzed the influence induced by magnetic field gradient in atom interferometer. The inhomogeneity of the magnetic field along the trajectory of atoms would induce differential quadratic Zeeman shifts for atoms in different paths, and this difference would induce a phase shift in the interference fringe. In 2007, Kasevich’s group reported a value of Newtonian constant of gravity measured via atom interferometer, they also proposed a scheme to test the charge neutrality of atoms. In 2009, Tino’s group studied the dispersion relation by atom interferometer, due to the applications of atom interferometer in atom recoil measurements, the dispersion relation at Planck-scale level can be studied in terrestrial laboratory. Schmidt et al. designed a portable atom gravimeter, it would be more precise than the best classical absolute gravimeter. That means the atom interferometer can be developed as commercial instrument in gravity field measurements.
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Since 1991, atom interferometer has been widely used in geodesy, inertial navigation, test of relativity, determination of fundamental constant and others. All these applications benefit from not only the special properties of cold atoms, but also the promotions of the techniques of atom interferometers. According to their features, interferometers can be classified as many categories, such as time domain or space domain interferometers, internal states or external states interferometers, near-field or far-field interferometers, and so on. But the most crucial part is the process of interference. In addition to promotion of related technologies, any methods that can enhance the divergence induced by external field will be helpful to improve the accuracy of the interferometer.

4. Gravity Measurement Using Atom Interferometry

Precision measurement of gravity is very important for geophysical research, seismology, resources exploration, environmental investigation, earthquake monitoring, metrology and fundamental science. A gravimeter is an instrument for measuring the local gravitational field. There are two types of gravimeters, relative gravimeters and absolute gravimeters. The most common relative gravimeters are spring-based gravimeters,61 and the most accurate relative gravimeters are superconducting gravimeters.62 Absolute gravimeters directly measure the acceleration of a mass during free fall in a vacuum;63 atom interferometric gravimeter51 is a new kind of gravimeter which uses microscopic atoms as test mass, it has better application prospects due to its intrinsic higher sensitivity.

4.1. Gravity measurement

Commercial absolute gravimeters measure the gravitational acceleration by monitoring the trajectory of a free fall mass. The mass includes a retro-reflector and terminates one arm of a Michelson interferometer. Portable absolute gravimeters have been widely used in gravity measurement due to their high accuracy and sensitivity. The first real atom gravimeter was demonstrated in 1999,51 since then, atom gravimeters have been continuously improved and enhanced. A lot of works have been done to decrease the measurement errors, and to promote the accuracy and sensitivity of atom gravimeters. The transverse motion of the atoms will induce the
parasitic acceleration. Besides Coriolis acceleration, the wave-front distortion is another effect relative with transverse motion of atoms. Bich et al. and Louchet-Chauvet et al. investigated the transverse motion of atoms in gravimeters, they tried to evaluate the effects by measuring the temperature of atoms. The retro-reflect configuration in atom gravimeters will cause an un-negligible delay for Raman beams, Nagornyi and his colleagues studied the delay effect and Doppler effect induced by finite speed of light, explained the discrepancy between theory and experiment.

Gravity field on Earth’s surface is not uniform, and gravity gradient is an important systematic error in gravity measurement. No matter how freely the atoms fall during measurement, the gravity gradient will influence the measurement results. Within a short distance, the gravitational field can be considered as a linear gradient field. Gravity gradient near Earth’s surface is approximately equal to $3 \times 10^{-6} \text{s}^{-2}$. If the falling height of atoms is more than 3.3 cm, the accuracy of the gravity measurement would not be better than $10^{-7} \text{m/s}^2$. Bertoldi proposed a method to solve this problem by compensating gravity, the gravity gradient within 10 cm scale can be flattened by a factor of 1000 after gravity compensation, and the effect of the gravity gradient can be ignored.

In 2011, Debs et al. used BEC in atom gravimeters, the free evolution time is 3 ms, and the accuracy is lower than $10^{-3}$. Hughes et al. used a different method in $^{87}\text{Rb}$ BEC gravimeter, equal interval laser pulses were used to suspend condensate, the information of the gravity can be obtained from the frequency of the laser pulse shining on the condensate. This method does not need space for free fall, the behavior of the atoms was consistent, and the suspension of the condensate can be hold for 100 ms. But the relative uncertainty of this measurement did not exceed $10^{-4}$ due to serious loss of atoms. Tino’s group measured the gravity with cold atoms in optical lattice, the measurement uncertainty is $10^{-7}$. They firstly decelerated strontium atoms with Zeeman slower, then trapped atoms in a blue detuned trap, and further cooled atoms in a red detuned trap. As an external force field, gravity will induce Bloch oscillations, and it can be measured by determining the period of Bloch oscillations. They compared the measurement results between this atom gravimeter and a commercial absolute gravimeter, and two results are consistent.

Portability is the prerequisite for the application of atom gravimeters. A mobile atom gravimeter was designed in 2009, the desired accuracy of
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this mobile atom gravimeter is $10^{-10}$. Tino’s group realized a portable laser system for high-precision mobile atom gravimeters. Merlet et al. designed and realized a mobile gravimeter for Watt Balance project, they demonstrated the comparison between atom interferometric absolute gravimeter and optical interferometric absolute gravimeter. Recently, Zhan’s group in China also realized a compact cold atom gravimeter, which measured the local gravitational acceleration.

4.2. Gravity gradient measurement

The gravity gradient is the spatial change of gravitational acceleration. Gravity gradiometers measure the spatial derivatives of the gravity vector; it is widely used in oil prospecting, mineral prospecting, water column density imaging, water depth determining, geophysics and inertial navigation. As mentioned above, cold atom interferometers can be used to measure gravity gradient.

Kasevich’s group measured the gravity discrepancy of two atom ensembles in 1998, which is the first atom gravity gradiometer. The schematic diagram of atom gravity gradiometer is shown in Fig. 2, the upper and lower atom ensembles separated by 1 m from each other, and these two atom gravimeters shared the same optical system. Data measured by absolute gravimeters can be used to determine vertical gravity gradients. In 2002, Kasevich’s group firstly used ellipse fitting method in coupled atom interferometers to extract phase difference, the induced gradient phase shifts can be rapidly extracted and gravity gradients can be accurately measured in the presence of common-mode vibration phase noise. They also demonstrated a absolute-gravity gradiometer, the corresponding sensitivity is $4E/\sqrt{\text{Hz}}$. Tino’s group presented the status of their high-sensitivity gravity-gradiometer based on atom interferometry, and the precision of $G$ determination may be below 100 ppm due to the high SNR and the long term stability of the gravity.

5. Rotation Measurement Using Atom Interferometry

Gyroscope is an instrument to measure changes in angle, or direction. Sagnac effect of interferometer was used in gyroscope in 1913, the path difference between two lights which propagated clockwise and counterclockwise respectively was used to measure the rotation of the
circular path, that is the principle of laser gyroscope and fiber gyroscope. Since the first demonstration of fiber interferometer, compact high-sensitivity optical gyroscope devices have rapidly developed. Sagnac effect of atom interferometer provides principle and technical basis for the development of atom gyroscopes. The performance of neutron interferometer in 1975 showed that matter wave interferometers can be used to measure the rotation. Atom gyroscopes will have higher sensitivity than optical gyroscopes due to lower velocity and shorter wavelength of atoms. Consider same area of the interference loop, the intrinsic sensitivity of atom gyroscopes may be 10 orders of magnitude higher than laser gyroscopes. In 1997, Gustavson et al. demonstrated a precision rotation measurement using an atom interferometer, the short term sensitivity of the atom gyroscope is $2 \times 10^{-8} \text{rad/s}/\sqrt{\text{Hz}}$. Similar to atom gravimeters, atom gyroscopes also take advantage of collective effect of single atom; the sensitivity of an atom gyroscope is limited by shot noise, which is inversely proportional to square root of the number of the atom. Dowling analyzed theoretically quantum-noise limits of atom gyroscopes, if the atoms are entangled, the

![Schematic diagram of atom gravity gradiometer.](image)
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sensitivity will exceed the shot noise limit. The gravity field induced effect cannot be distinguished by atom gyroscope. Kasevich’s group designed a dual loop atom gyroscope in 1998 to decrease gravity, the Sagnac phase shifts of the counter propagating atoms are opposite, but the gravity induced phase shifts will be the same, measuring the differential phase shifts will reject the gravity induced systematic error, so the sensitivity of the dual loop atom gyroscope is expected to be improved at least one order of magnitude. They demonstrated an improved sensitivity of $6 \times 10^{-10} \,(\text{rad/s})/\sqrt{\text{Hz}}$ in 2000.

The characteristics of atom gyroscopes and atom gravimeters are quite similar, such as phase noise due to vibrations. A double-loop atom interferometer can be used to measure either gravity or rotation. However, there are still some differences between atom gyroscopes and atom gravimeters. For an atom gravimeter, both the free falling direction of atoms and the propagating direction of Raman beams are parallel to gravitational field, and the scale factor is proportional to time interval between Raman pulses. While for an atom gyroscope, the angle between propagating direction of Raman beams and atoms will gradually change due to gravity, and the scale factor is proportional to distance interval between Raman pulses. As to atom gyroscope, if one only considers the pros and cons of splitting, microstructure seems to be a better splitter; Xia et al. demonstrated a nanometer scale transmission gratings for atom interferometric gyroscope applications.

Diamagnetic force was used in gyroscope in 2001. Although this method was used to improve the gravity induced drift in mechanical gyroscopes, it is also possible to produce a continuous micro-gravity environment for atom gyroscopes, and thus the systematic errors associated with gravity will be greatly reduced. The sensitivity of atom gyroscopes can be enhanced by increasing the number of atoms or increasing the scaling factor, $2k_{\text{eff}}L^2/v \sin \theta$, where $k_{\text{eff}}$ is effective wave vector of Raman lasers, $L$ is the distance interval of Raman pulses, $v$ is the velocity of atoms, and $\theta$ is the angle between rotating direction and atom propagating direction. The short-term sensitivity of an atom gyroscope is limited by the shot noise, and the long-term sensitivity is limited by the optical aberrations. The high-flux atom beam can be achieved, and the highest sensitive atom gyroscope was realized by thermal atom beams, which have much more atoms involved in measurement.
Compact cold atom gyroscopes should have higher sensitivity than thermal atom beam gyroscopes, but a relatively small number of atoms involved in measurement reduce the SNR of the interference fringes. The cold atom gyroscope was realized in 2006 with nearly $10^7$ atoms, and the achieved sensitivity was poor.\textsuperscript{92} Increasing the number of cold atoms is an important trend for the future development of cold atom gyroscope. In 2007, a high-flux cold atom beam for atom gyroscope was demonstrated.\textsuperscript{93} Two-dimensional (2D) MOT was used to generate continuous low speed atom beam, and to increase the loading rate of the three-dimensional (3D) MOT, and such a system can provide at least $10^{10}$ atoms per second. Recently, high-flux BEC\textsuperscript{94} and superfluid\textsuperscript{95} are used as atom sources for gyroscopes.

Gyroscopes can be used for determination of latitude and azimuth, and inertial navigation. The miniaturization of gyroscope is very important for its application. Most of present atom gyroscopes are laboratory prototypes; bulky vacuum system, heavy magnetic shielding and complex optics are difficult to miniaturize. Cold atoms are more suitable than thermal ones for compact atom gyroscopes. In 2009, a compact dual-loop cold rubidium atom gyroscope was realized\textsuperscript{96} by Muller \textit{et al.}, the schematic diagram is shown in Fig. 3. They designed aluminum 2D-MOTs and 3D-MOTs chambers, a 20l/s ion pump and a titanium sublimation pump were used for obtaining high-vacuum background. Cold atom beams were generated by 2D-MOTs,
atom numbers of which were comparable with the thermal atom beams. The size of the vacuum system is $120 \times 90 \text{ cm}^2$, although the volume of the whole device is still big, its portability is better enough for field survey. In practical field survey, an atom gyroscope must match the requirements of dynamic measurement, for this purpose, a shorter measurement circle is necessary. In 2011, Stoner’s group demonstrated an atom gyroscope with short interrogation time of millisecond, the interaction between Raman lasers and atoms, and the detection of population are completed in MOT region, and the fringe contrast was better than expectation. They found that the measurement is limited by the intensity noise of Raman lasers, and investigated carefully the dependence of measurement process on laser intensity noise. Kasevich’s group demonstrated an improved atom gyroscope, they overcame the accuracy and dynamic range limitations of previous atom gyroscopes, achieved a sensitivity of $2 \times 10^{-12} \text{ rad/s per shot.}$

6. Gravitational Wave Detection

6.1. Gravitational wave

General relativity predicts that the accelerated motion of the mass will generate gravitational waves. Gravitational waves have following properties: they propagate at the speed of light in vacuum; carrying energy and information of wave sources; the absorption rate by materials is very low; having two independent polarization states. The energy that gravitational waves carry makes them detectable. But due to the very weak strength of the gravitational wave, direct observation of gravitational waves is very difficult. The detection of gravitational waves is of great scientific significance. Once the gravitational wave is proved to really exist, it will open a brand new page for astronomy and physics.

The sources of gravitational waves can be classified as three kinds. The first kind is pulsed gravitational wave which is generated when universe incidents happen. The second is frequency stable gravitational wave which is generated by binary system or neutron star spinning. The third is the irregular gravitational wave from cosmic background. For more than 40 years, physicists have not found any proof of gravitational waves, until Taylor and his partners calculated the period of the binary pulsar, named as PSR1913 + 16, using data of 18 years’ continuous observation, their
calculation in 1975 agreed well with the theoretical prediction, and their experiments indirectly proved the existence of gravitational waves and cheered the scientists who are dedicated to the detection of gravitational waves.

6.2. Situation and progress

One kind of gravitational wave detectors is Weber detector, which detect gravitational waves through measuring the variation of distance between two objects. There are five Weber detectors around the world, two of them are in Italy, and the other three are in Switzerland, the United States of America (USA) and Australia respectively. Due to the small size of their antennas, Weber detectors can only detect high frequency gravitational waves, which could only be generated by rare sudden cosmic events. The waiting type experiments and continued operation of cryogenic system need expensive costs to support such research. In the middle of 1970s, scientists started to use laser interferometers as antennas to detect gravitational waves, the antennas is similar to Michelson interferometer, and a pair of mirrors at each arm is used to form a Fabry–Perot cavity. Multiple reflections can increase the optical path difference caused by variations of mirrors’ position. There are six high-sensitivity laser gravitational wave detectors in the world. Among them, Virgo is a cooperation project of Italy and France with two 3-km-long arms and its measurement bandwidth covers 10 Hz to 3 kHz. Laser interferometer gravitational-wave observatory (LIGO) were built and operated in two places of USA, one interferometer’s arm is 4 km long, and the other is 2 km long, two interferometers can simultaneously detect high frequency gravitational waves. GEO 600 was built in Germany, and it is a joint project of British and German, the arm length is 600 meters. TAMA detector was built in Japan; the baseline is 300 m. Australian consortium for interferometric gravitational astronomy (ACIGA) is the only laser gravitational-wave detector built in the southern hemisphere, its arm length is only 80 m, but it can be extended with high optical power in resonant cavity. Above six laser gravitational-wave detectors can only detect the gravitational waves with frequency from $10^{-2}$ to $10^4$ Hz. National Aeronautics and Space Administration (NASA) and European Space Agency (ESA) are joining to promote a laser interferometer space
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antenna (LISA) for gravitational-wave detection.\(^{111}\) Three ground-based detectors are respectively set in three satellites which will form an equilateral triangle with a side length of \(5 \times 10^6\) km. The regime of the gravitational wave can be detected by LISA is \(10^{-4}\) to \(10^{-1}\) Hz.

Although, so many efforts have been taken, scientists still do not get direct evidence for gravitational waves. The development of the atom interferometer provides the inspiration for gravitational wave detection. In 2004, Chiao and Speliotopoulos proposed a scheme for matter-wave interferometric gravitational-wave detector,\(^{112}\) further cooled supersonic atoms beam will be used as matter wave sources, the size of this type detector will be much smaller than LIGO type detector, and the detectable bandwidth of gravitational waves will be wider. Matter-wave interferometric gravitational-wave observatory (MIGO) will become a new generation of gravitational wave detectors.

6.3. Gravitational wave detection using atom interferometer

Since 1979, scientists had already started to investigate the interaction between matter waves and gravitational waves.\(^{113}\) Chiao claimed that for the same size devices, MIGO type detectors' sensitivity would be several orders of magnitude higher than LIGO type detectors,\(^{112}\) but others did not think so.\(^{114}\) In 2007, Tino and Vetrano carefully analyzed the possibility of atom interferometer based gravitational wave detectors,\(^{115}\) the phase shifts for different shape interferometers were discussed. They found that MIGO can be used to detect low frequency gravitational waves, MIGO has comparable sensitivity with laser gravitational wave detectors while it does not need so huge device sizes. It seems that MIGO is a potential detector which can compensate the LIGO’s defects. In 2011, Hohensee et al. listed sources and technology for MIGO, the frequency band of gravitational wave that MIGO can sensor is at \(\text{mHz} \sim \text{Hz}\) level.\(^{116}\) In 2009, Ambrosio analyzed major noises of MIGO,\(^{117}\) and he found that the main limitation is the noise of the atom interferometers. Charged molecule was proposed as test mass for gravitational wave detector in 2010 (Ref. 118) by Lorek et al., molecules were not spatial split, but oriented in space along the two polarization direction of gravitational waves, and the detector will have a higher sensitivity. In 2011, Lorek et al. further analyzed influence of a monochromatic gravitational wave over a quarter period,\(^{119}\) this scheme
can be extended to an atom interferometer; Zhan’s group proposed a new gravitational wave detection scheme which combined Chiao’s and Kasevich’s proposes, standing waves instead of material gratings will be used to split, reflect and recombine atoms, supersonic atom beams will be used to improve the SNR. This device would be suitable to detect the gravitational waves with frequency range from 10 to 1000 Hz. Another scheme with configuration similar to LIGO was proposed, two atom interferometers, separated by a distance, act the role of the mirrors in LIGO.

A space project on atom interferometer based gravitational wave detector was proposed in 2009 (Ref. 122), this detector will probe gravitational waves with the same frequency range as LISA and with similar sensitivity, in addition to space plan, a terrestrial 10 m atom interferometer project was proposed. In 2011, a comment on atom type gravitational wave detectors was reported, error sources that had not been concerned before was further investigated. Another atomic gravitational wave interferometric sensor in low earth orbit (AGIS-LEO) was proposed in 2011 (Ref. 124), three atom interferometers will be placed in space with 30 km distance between each of pair, and the gravitational waves will be detected by comparing any pair of the atom interferometers. This project is designed for detecting the gravitational waves with frequency range from 50 mHz to 10 Hz, and the sensitivity will be $10^{-18}/\sqrt{\text{Hz}}$.

It seems that, atom interferometers have higher sensitivity than laser interferometers in detection of gravitational waves with special frequency band, and construction of atom interferometer based gravitational-wave detectors is necessary. The network of LIGO and MIGO will make a significant contribution to directly prove the existence of gravitational waves.

7. Cold Atom Clock

7.1. Microwave clock

In early days, quartz oscillators, which had fractional frequency instabilities of $2 \times 10^{-8}$, was used to define the second in the International System of Units (SI). Quartz oscillators are sensitive to temperature, and are individually different from each other; their frequencies usually are not equal and stable. The development of the microwave spectroscopy makes
usage of unperturbed atomic transitions as frequency standards become a reality. In atom beam experiments, Doppler shifts and limited interaction times restrict both the accuracy and stability of the frequency measurement. To solve these problems, Zacharias\textsuperscript{126} proposed a fountain type frequency standard in 1953. Unfortunately, the fountain did not work well. In order to obtain sufficiently strong signal, laser-prepared atomic fountains for optical frequency standards applications were proposed in 1989 by Hall \textit{et al.}\textsuperscript{127} Rf spectroscopy was used in fountains by Kasevich \textit{et al.}\textsuperscript{128} A lot of primary clocks are built gradually due to the improvement of key technologies of frequency standard based on atomic transitions. Since the late 1990s, some atom fountains were established successively as primary clocks,\textsuperscript{129} such as NIST-F1 at the National Institute of Standards and Technology (NIST) in USA, CSF1 at Physikalisch-Technische Bundesanstalt (PTB) in Germany, CsF1 at the Istituto Elettrotecnico Nazionale (IEN) in Italy, CsF1 at the National Physical Laboratory (NPL) in UK, FO2 and FOM at the Observatoire de Paris in France. Other laboratories are developing atom fountains for fundamental physics experiments. Ovchinnikov and Marra reported new progress on rubidium atomic fountain of NPL in 2011,\textsuperscript{130} Fig. 4 shows a simplified setup of the atomic fountain at NPL. Like most other fountain clocks, it consists of four parts: MOT, state-selection system, the interrogation system and detection system. But the difference is that it has two MOTs, the cold $^{87}\text{Rb}$ atoms in the main MOT are all coming from the other low-velocity intense source (LVIS). This novel design not only traps more $^{87}\text{Rb}$ atoms, but also can get rid of all hot $^{87}\text{Rb}$ atoms in the vacuum chamber as well as the high abundance useless $^{85}\text{Rb}$ isotope atoms. Two other features are water-cooled temperature-stabilized interrogation region and high quality factor ($Q_c \approx 28500$) interrogation cavity. The fractional frequency accuracy of the atom clock is estimated to be $3.7 \times 10^{-16}$ as shown in Table 1, the stability is limited by local oscillator which is estimated to be $7 \times 10^{-16}$ in one day’s average. Another primary frequency standard at NPL, the NPL-CsF2, also made a good progress in 2011. They evaluated the two major sources of uncertainty for the NPL-CsF2 cesium fountain clock, the distributed cavity phase (DCP) and microwave lensing frequency shifts,\textsuperscript{131} the DCP uncertainty is reduced to $1.1 \times 10^{-16}$ and the type B uncertainty of the NPL-CsF2 clock is reduced to $2.3 \times 10^{-16}$. Clairon’s group at Observatoire de Paris also did a great work on evaluating the DCP uncertainty,\textsuperscript{132}
they improved their DCP uncertainty to $\delta v/v = \pm 8.4 \times 10^{-17}$. In addition, they demonstrated that by using a cavity with four independent, azimuthally distributed feeds, the DCP uncertainty will probably be reduced to less than $\pm 1 \times 10^{-17}$.

Weyers’ group also evaluated the frequency error from DCP and the frequency bias and uncertainty due to the microwave lensing of the atomic wavepackets in the cesium fountain clock PTB–CSF2 at PTB, they reported a total systematic uncertainty of $4.1 \times 10^{-16}$. Dube’s group made an accurate map of the C-field in the cesium fountain frequency standard FCs1 at National Research Council of Canada (NRC), as they expected that
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Table 1. Frequency shift and their type B uncertainties.\(^{130}\)

<table>
<thead>
<tr>
<th>Effect</th>
<th>Frequency shift (10^{-16})</th>
<th>Uncertainty (10^{-16})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Second order Zeeman</td>
<td>1922.3</td>
<td>1.0</td>
</tr>
<tr>
<td>Blackbody radiation</td>
<td>-133.5</td>
<td>0.6</td>
</tr>
<tr>
<td>Cavity distributed phase</td>
<td>0</td>
<td>3.0</td>
</tr>
<tr>
<td>Cavity pulling and collisions</td>
<td>-0.6</td>
<td>0.2</td>
</tr>
<tr>
<td>Collisions with background gas</td>
<td>&lt;0.7</td>
<td>1.0</td>
</tr>
<tr>
<td>Microwave leakage</td>
<td>0.55</td>
<td>1.0</td>
</tr>
<tr>
<td>Gravity</td>
<td>12.6</td>
<td>0.3</td>
</tr>
<tr>
<td>Other</td>
<td>-</td>
<td>1.0</td>
</tr>
<tr>
<td>Total uncertainty</td>
<td>—</td>
<td>3.7</td>
</tr>
</tbody>
</table>

the inhomogeneity of the magnetic field should give rise to one of the main sources of uncertainty, and contribute to make the second-order Zeeman shift better than the \(10^{-15}\) level. Ido’s group reported the total uncertainty of the NICT–CsF1 at the National Institute of Information and Communications Technology (NICT) in Japan, and it was evaluated as \(4.1 \times 10^{-15}\), they also successfully observed the Ramsey fringe with a linewidth of 0.95 Hz in NICT-CsF2 and its instability is \(5 \times 10^{-13}/\sqrt{\tau}\).\(^{135}\) Li’s group at National Institute of Metrology (NIM) in China moved and recovered Cs fountain clock, the preliminary results show a stability of \(8 \times 10^{-16}\) over 1000 s averaging time.\(^{136}\)

Müller et al.\(^{137}\) demonstrated a novel compact frequency standard. Their experiment setup and temporal working cycle is shown in Fig. 5. As it is shown on the left side of Figure 5, the whole equipment is just a little bit bigger than its microwave cavity, which is wholly made of stainless steel. The trapping, repumping, interrogation and detecting procedures are all carried out in the same position as it shows on the right side of Fig. 5.\(^{137}\) There are many advantages of all-in-one configuration in a compact cold atom frequency standard, such as compact structure, small weight and better magnetic field shielding. Its stability could reach up to \(\sigma_c(\tau) = (5 \pm 0.5) \times 10^{-13}/\sqrt{\tau}\) for the clock transition with a linewidth of 47(\(\pm\)5) Hz.

A new principle of microwave frequency standard is coherent population trapping (CPT). CPT clocks have characteristics of small size, low power consumption and good long-term performance. Cold atom CPT clocks will be good candidates for commercial and spatial applications.
Zhan’s group reported an experimental observation of coherent population trapping-Ramsey interference in cold $^{87}\text{Rb}$ atoms.\textsuperscript{138} Esnault\textit{ et al.}\textsuperscript{139} designed a compact cold-atom CPT frequency standard, their expected frequency stability should be better than $10^{-11}/\sqrt{\tau}$ and an accuracy will be around $10^{-13}$.

### 7.2. Space clock

Frequency comparisons and fundamental physics in space acquire high accuracy clocks. Microgravity environment is conducive to improve the performance of atom clocks. Scientists want to launch atom clocks to space for scientific and technology purpose, some space clock projects have proposed, such as ACES (Atomic Clock Ensemble in Space)\textsuperscript{140} and HORACE (Clock with atom cooling in a cell).\textsuperscript{141}

The payload of ACES accommodates two atom clocks, PHARAO (a primary frequency standard based on samples of laser cooled cesium atoms) and SHM (an active hydrogen maser for space applications). PHARAO will have fractional frequency instability of $10^{-13}/\sqrt{\tau}$ and an inaccuracy of few parts in $10^{16}$; SHM will have fractional frequency instability of $1.5 \times 10^{-15}$ after 10,000 s integration. The short-term stability of SHM and the long-term stability of PHARAO will be used to perform comparisons.
of atom frequency standards. Ultimately, researches on general relativity, gravitational redshift, time variations of fundamental constants and so on will be carried out through precision measurements and comparisons of frequencies. Nearly all the components of ACES have been prepared and tested, it will be launched to International Space Station (ISS) between 2013–2014.

HORACE is a compact cold cesium atom clock developed at Paris Observatory. It has a relatively small size because of performing the whole measurement procedures at the same place by using the isotropic light cooling techniques. As a candidate for Galileo’s ground segment clocks and onboard Galileo clocks, HORACE has short-term relative frequency instability of $2.2 \times 10^{-13}/\sqrt{\tau}$ and long term instability of about $3 \times 10^{-15}$ in $10^4$ s integration according to current results.

Nissen et al. proposed a novel microwave clock for space purpose, it is suitable for relativity experiments on Earth orbit, and its goal is to reach a frequency discrimination level of about $1 \times 10^{-16}$ in a single rotation of the satellite roll period which is about 1000 s. Bandi et al. proposed another space clock, the expected short-term stability is $4 \times 10^{-13}/\sqrt{\tau}$ for integration time 1 s–1000 s, and a medium-to long-term stability is at $1 \times 10^{-14}$ level.

### 7.3. Optical clock

Nowadays, optical clocks based on single ions and neutral atoms have surpassed the primary microwave frequency standards either on stability or accuracy, and have a remarkable potential for further developments. The theoretically achievable fractional frequency instability of the optical clocks can be scaled as,

$$\sigma_f(\tau) \propto \frac{1}{Q(S/N)}\sqrt{\tau}, \quad (1)$$

where, $Q$ is the line quality factor (the ratio of the transition frequency $v_0$ to its linewidth $\Delta v_0$), $S/N$ is the SNR for a 1 Hz detection bandwidth, and $\tau$ is the averaging time in seconds. From this equation, one can see that changing from a microwave transition to an optical transition can lead to an immediate increase in stability. At the meantime, the reproducibility and interference immunity to environmental disturbances, such as electric or magnetic field,
are also needed to be considered in frequency standards. Obviously, optical frequency standards can meet the qualification. The candidates for optical frequency standards can be classified as single trapped ions and cold neutral atom cloud. A simplified diagram of a optical clock is shown in Fig. 6.146

Single trapped ions are well isolated from environmental perturbations, and thus have narrow linewidth transitions. The future high accurate optical frequency standards or clocks will use single trapped ion techniques. Dubé demonstrated the advancement of strontium ion optical frequency standard in NRC,134 they designed an end cap trap which minimized micro motion in three dimensions, obtained a stability of $6.5 \times 10^{-16}$ at 500 s, and a frequency uncertainty of $5 \times 10^{-17}$ after one day’s averaging. Peik’s group147 at PTB has demonstrated that the $^2S_{1/2} \rightarrow ^2F_{7/2}$ octupole transition in $^{171}$Yb$^+$ can be used to realize an optical clock with a systematic uncertainty of $7.1 \times 10^{-17}$.

Equation (1) shows that the instability is inversely proportional to SNR, while SNR is proportional to square root of atom number, $\sqrt{N}$; frequency standards using large numbers of cold atoms potentially have
higher stability than that of using single trapped ions. That is why so many groups construct optical frequency standards based on cold neutral atom ensembles prepared in MOT, optical lattice and continuous beam. Hansch et al.\textsuperscript{148} have measured the $1S \rightarrow 2S$ transition frequency in hydrogen atoms via two-photon spectroscopy, they obtained the value $f_{1S \rightarrow 2S} = 2\,466\,061\,413\,035(10)$ Hz, a fractional frequency uncertainty is $4.2 \times 10^{-15}$. Katori used the optical lattice to confine the cold atoms during the clock interrogation cycle in 2001; this greatly prompted the application of optical lattice technique in time and frequency standards. Katori reviewed the experimental realization, optimal design, and future applications of neutral atom optical clocks.\textsuperscript{149} We know that, as the number of atoms increases in optical lattice, the collision frequency shifts also increases. In 2011, Ye’s group proposed a solution for this problem by dramatically increasing the strength of atom interactions,\textsuperscript{150} by suppressing collision shifts in lattice sites containing atoms ($N > 1$), strong interactions introduced an energy splitting and an evolution to a many-particle state in which collisions is restrained. They demonstrated that the uncertainty of strontium lattice optical clock is $10^{-17}$. Bishof et al. resolved atomic interaction sidebands in $^{87}\text{Sr}$ optical clock transition,\textsuperscript{151} observed two-body loss of $3\, P_0$ $^{87}\text{Sr}$ atoms trapped in a one-dimensional optical lattice and measured loss rate coefficients of atoms.\textsuperscript{152} Lemke et al. studied ultracold collisions in Fermionic Ytterbium,\textsuperscript{153} Ludlow et al. demonstrated that the cold-collision shift can be cancelled at the $5 \times 10^{-18}$ level in a $^{171}\text{Yb}$ optical lattice clock.\textsuperscript{154} Yi et al. found that the magic wavelength for the clock line in neutral Hg is 362.53 (0.21) nm.\textsuperscript{155} Westergaard et al. presented the frequency shifts associated with the lattice potential in a Sr lattice clock by comparing two such clocks with a frequency stability of $5 \times 10^{-17}$.\textsuperscript{156} Falke et al. reported their strontium optical frequency standard contributes with a fractional uncertainty of $1.5 \times 10^{-16}$.\textsuperscript{157}

Laser stabilization is one of the key techniques in optical frequency standards. Ludlow’s group demonstrated a cavity-stabilized laser system; it has been used as a stable optical source in an ytterbium optical lattice clock to resolve an ultra-narrow 1 Hz transition.\textsuperscript{158} Tino’s group used two-stage frequency stabilization techniques to control high-finesse optical cavities,
the laser frequency noise they measured is $2 \sim 11 \text{ Hz}$, that is about three time of the cavity thermal noise.159

Femtosecond optical frequency comb is another key technique in optical frequency standards. In 2011, Kippenberg et al. presented a new optical frequency comb which used parametric frequency conversion in high resonance quality factor micro resonators,160 this approach not only causes a dramatic reduction in size but also has access to comb generators with a repetition rate up to $\sim 1 \text{THz}$.

Comparisons are feasible methods to determine the reproducibility of frequency standards. Long distance frequency comparisons are performed via satellites with a fractional instability of around $10^{-15}$ for a measurement time of one day. But with fractional instabilities of around $10^{-15}$ at 1 s, optical frequency standards are not appropriate for the satellite-based methods. Therefore, transferring stable optical frequency by optical fibers is needed. Recently, Fujieda et al. developed an all-optical link system which has the instability less than $2 \times 10^{-15}$ at 1 s and $7 \times 10^{-17}$ at 1000 s.161 they performed a $^{87}\text{Sr}$ based optical lattice clock comparison between NICT and University of Tokyo with the fractional instabilities of around $7.3 \times 10^{-16}$.162 Katori’s group proposed a novel method which made frequency comparison through synchronously manipulating different isotopes such as $^{87}\text{Sr}$ (Ref. 149) and $^{88}\text{Sr}$ (Ref. 163), the relative stability they achieved is $3 \times 10^{-17}$.

### 7.4. Chip based clock

Compact and portable frequency standards have potential application in scientific research and technology in future. Atom chips could be an excellent carrier to develop the compact high stability clocks. Ramírez-Martínez et al. presented a chip based $^{87}\text{Rb}$ atom frequency standard, the frequency stability is $\sigma_f = 1.5 \times 10^{-12} \sqrt{\tau}$ for $10 < \tau < 10^3 \text{s}$.164 Farkas et al. demonstrated a chip based atom clock that used all-optical interrogation of ultracold rubidium atoms, the clock inaccuracy is better than $2 \times 10^{-14}$.165 Although the performances of these chip based clocks, such as the stability, and the shot-noise-level, are not as good as previous clocks, they still have broad application prospects in the future.
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8. Determination of the Fundamental Constant

8.1. Determination of gravitational constant

Newtonian gravitational constant which is known as “capital G” is closely related to theoretical physics, astrophysics and geophysics. The experimental measurement of the gravitational constant can be traced back to the late 18th century. Gravitational effect is weak and cannot be isolated and shielded. It is so difficult to measure experimentally that after more than 200 years’ efforts, the measurement precision of this constant has only been improved by two orders of magnitude.

8.1.1. Traditional methods of gravitational constant measurement

In 2006, the recommended value of $G$ by the Committee on Data for Science and Technology (CODATA) is $6.67428(67) \times 10^{-11} \text{m}^3\text{kg}^{-1}\text{s}^{-2}$, and the relative uncertainty is $1.0 \times 10^{-4}$. In 2010, the fifth recommended value of $G$ is $6.67384(80) \times 10^{-11} \text{m}^3\text{kg}^{-1}\text{s}^{-2}$, and the relative uncertainty is $1.2 \times 10^{-4}$. We find that, the new recommended measurement precision of $G$ becomes better, but relative uncertainty of $G$ turns worse. Not only the values measured by different methods disagreed with each other, but also the values measured by the same methods at different time disagreed with each other. And this worse disagreement made the new recommended value have larger expansion.

Two kinds of methods used to measure $G$ are torsion balance and beam balance. The group at University of Washington measured $G$ with fiber torsion balance, the value they measured in 2000 is $6.674255(92) \times 10^{-11} \text{m}^3\text{kg}^{-1}\text{s}^{-2}$. The group at University of Zurich measured $G$ with beam balance, the value they measured in 2006 is $6.67425(12) \times 10^{-11} \text{m}^3\text{kg}^{-1}\text{s}^{-2}$.

Both torsion balance and beam balance used macroscopic objects as test mass, microscopic particles was not used in $G$ measurement until atom interferometry was applied in precision measurement.

8.1.2. Determination of gravitational constant with cold atoms

Cold atoms can be used in high sensitivity gravity measurement and have potential to determine the Newtonian gravitational constant. In
2003, Tino’s group proposed a feasible scheme to measure \( G \) for an expected accuracy of \( 10^{-4} \).\(^{171}\) two atom clouds were launched and formed a configuration of gradiometer.\(^{50}\) To reach the accuracy of \( 10^{-4} \), the gravity measurement must achieve accuracy of \( 10^{-11} \) g with single atom cloud launched scheme, which seems difficult. Differential measurement will greatly suppress common mode noise and thus increase the accuracy of gravity gradient measurement, heavy source masses can be used to change the local gravitational acceleration, this method is similar to beam balance, two atom clouds are test masses, and the source masses are field masses. Atom interferometric method has two advantages: the properties of certain atoms are well known; and the gravity variation can be directly measured without any other gravimeters. While, characterizing the source masses and precise positioning of the test mass is still a problem.

In 2007, the first value of \( G \) measured by cold atom interferometer was reported by Kasevich’s group.\(^{57}\) The schematic of experiment is shown in Figure 7; a 540 kg lead was used as the source mass which can move along the vertical direction. Upper and lower gravimeters share the same Raman lasers. The inferred value of \( G \) is \((6.693 \pm 0.027) \times 10^{-11} \) m\(^3\)kg\(^{-1}\)s\(^{-2}\), with a relative uncertainty of \( 4.0 \times 10^{-3} \). Similar to beam balance method, the limitation of measurement accuracy of this method are determination of initial location, velocity of atoms and positioning of the source mass. In 2008, Tino’s group determined a new value of \( G \),\(^{172}\) two high density tungsten source masses were used to induce gravitational field, and their gravity gradiometer was formed with successively launched atom ensembles with different launch heights. The value of \( G \) they measured is \( 6.667 \times 10^{-11} \) m\(^3\)kg\(^{-1}\)s\(^{-2}\) with a relative uncertainty of \( 1.6 \times 10^{-3} \). Until now, only two groups have experimentally determined the \( G \) using cold atoms.

Recently, Tino’s group upgraded their atom preparation system and Raman laser system,\(^{78}\) a 2D-MOT was used to provide high flux atom beam, which can promote the loading rates and preserve a low background pressure. A new Raman lasers system was used, which has lower intrinsic frequency noise with an output power of 1 W. The measurement uncertainty under 120 hours’ integration is \( 4.0 \times 10^{-4} \).

Within just eight years, the measurement precision of \( G \) using cold atoms has reached a comparable level with the traditional measurements’. Although the accuracy of atom interferometric method has not been
accepted by CODATA yet, there is still great space for this new method to optimize and to improve.

8.2. Determination of fine structure constant

8.2.1. Traditional methods of fine structure constant determination

The fine structure constant, $\alpha$, is a dimensionless constant and appears in different domains of physics, its value can be measured by three methods. The first method is combination of electron anomaly measurement and quantum electrodynamics (QED) calculation, the second is measuring the quantum Hall effect, and the third is measuring the ratio of Plank constant
and atomic mass. The most precision value of $\alpha$ was determined by the first method, and the most precision measurement using electron anomaly was realized by Gabrielse’s group at Harvard University,\textsuperscript{173} combining this measurement and the QED calculation, the determined value of $\alpha^{-1}$ is 137.035999084, with a relative uncertainty of $3.7 \times 10^{-10}$. The most precision value of $\alpha$ determined by quantum Hall effect is 137.0360037, with a relative uncertainty of $2.4 \times 10^{-8}$.\textsuperscript{174} The most precision value of $\alpha$ determined by the ratio of Plank constant and atomic mass is 137.035999037, with a relative uncertainty of $6.6 \times 10^{-10}$.\textsuperscript{175} And the recommended value of $\alpha^{-1}$ by CODATA in 2010 is 137.035999074, with a relative uncertainty of $3.2 \times 10^{-10}$.\textsuperscript{176}

8.2.2. Determination fine structure constant with cold atoms

For a long period of time, the fine structure constant was mainly determined by the method of combination of electron anomaly measurement and QED calculation. The determination using only one kind of methods is unreliable, especially the determination of $\alpha$ had been shifted because of a mistake of the calculation. So other methods that are independent on QED are needed. In turn, comparing the values of $\alpha$ inferred from different methods can test QED theory. A well known relationship on determination of fine structure constant is,

$$\alpha^2 = \frac{2R_\infty}{c} \frac{h}{e} \frac{m_a}{m_e},$$

(2)

where, $R_\infty$ is the Rydberg constant, $c$ is the speed of light, $m_a$ is the mass of atom, $m_e$ is the mass of electron, and $h$ is Planck constant. The last term of the right side of the equation decides the precision of $\alpha$ so far. Due to the relationship, $v_r = \frac{\hbar k}{m_a}$, $v_r$ is the recoil velocity, $\hbar$ is reduced Plank constant, $k$ is the wave vector of light, one can infer the last term in Eq. (2) by measuring the value of recoil velocity.

In 1994, Chu’s group measured the velocity variation of cesium atom,\textsuperscript{48} they carefully analyzed the errors during measurements, and obtained the value of $h/m_{C_s}$ ($m_{C_s}$ is the mass of cesium atom), the disagreement with well known value of $\alpha$ is 0.1 parts per million. They improved their experiments in 2002, and determined that the value of $\alpha^{-1}$ is 137.036000, with a relative uncertainty of $7.4 \times 10^{-9}$.\textsuperscript{176} In 2006, they proposed a new feasible
scheme to measure $\alpha$ with an expected precision of $5 \times 10^{-10}$, the Bragg diffraction will be used instead of the stimulate Raman transition, atoms would stay in the same internal states which can avoid the influence induced by differential of the internal states; two conjugate interferometers will be used in to reduce some systematic errors, especially the gravity induced phase shift. In addition, lower noises during the measurements, enhance the velocity variation due to photon kicking will intrinsically promote the precision of the measurements. In 2008, they improved the precision of measurement of $\alpha$ to $3.7 \times 10^{-10}$. Recently, the atom’s momentum can be changed up to 102 photon recoil momentums, this would greatly help to determine the value of fine structure constant.

Another group also measured the value of $\alpha$ with cold atoms, two conjugate interferometers were also used to detect the velocity variation of the atom, and Bloch oscillations were used to split atoms, which can efficiently transfer large number of photon recoil momentums. In 2008, they achieved 1600 photon momentums transfer, the more momentum transfer was limit by their device’s size. The determined value of $\alpha^{-1}$ is 137.03599945, with a relative uncertainty of $4.6 \times 10^{-9}$. In this experiment, atoms keep oscillating in laser induced potential field, so the requirements for laser quality are very strict. The experiment results are limited by wave front curvature, frequency stabilization of lasers and the alignment of laser beams. They obtained an improved value of $\alpha^{-1}$, which is 137.035999037, with a relative uncertainty of $6.6 \times 10^{-10}$, this is the most accurate value of fine structure constant inferred by cold atoms.

Until now, two groups measured the value of $\alpha$ with precision of $10^{-10}$ by cold atoms. This may totally change the situation of determination of $\alpha$, and provide a possibility to stringently test QED theory. Larger area interferometer will be used to promote the precision, and the uncertainty of $m_\alpha/m_e$ will be the major limitation of determination of $\alpha$ with cold atoms.

9. Testing Fundamental Physics

Einstein’s equivalence principle (EP) is a cornerstone of general relativity, and EP contains three assumptions: (1) the universality of free fall (UFF), known as weak equivalence principle (WEP), which states that freely falling bodies have the same acceleration in the same gravitational field
independent of their compositions; (2) Local Lorentz invariance (LLI), which assumes that clock rates are independent of the clock velocities; (3) Local position invariance (LPI), which assumes that clock rates are independent of their space-time positions. Some theory predicted the possibility of the violation of EP. The violation of the EP seems to be the requirement of the unified theory. This situation greatly inspired physicists to further test EP. Usually the test of EP starts from the test of WEP.

9.1. Equivalence principle test

The WEP states that the gravitational mass is equal to inertial mass. Usually it is tested by comparing the gravitational acceleration of different free falling test bodies. The earliest test of WEP can be traced back to the famous Galileo’s leaning tower experiment,\(^{181}\) that experiment kindly proved the WEP, but the precision was quite low.

Eötvös suspended two balls which were made of different materials on two sides of a torsion balance. The two balls were affected not only by gravitational force, but also by the inertial centrifugal force. If the WEP was violated, the torsion of the balance would not be zero. Eötvös’ experiment greatly improved the precision of WEP test. To commemorate his contribution to WEP test, a dimensionless Eötvös-parameter, \(\eta\), is used to scale the precision of the WEP test,

\[
\eta = \frac{2(a_1 - a_2)}{a_1 + a_2},
\]

where, \(a_1\) and \(a_2\) are the acceleration of the two test masses respectively.

The most precision test of WEP with a torsion balance was reported in 2008, the precision of \(\eta\) is \(10^{-13}\).\(^{182}\) Another method to test WEP, Lunar Laser-Ranging (LLR), also achieved the precision of \(10^{-13}\) (Ref. 183), in the Sun–Earth–Moon system, the Earth and the Moon can be considered as two test masses which are free falling to the Sun. With new APOLLO facility, the precision of LLR will reach millimeter level range, and that will lead to a test of WEP at the level of \(10^{-14}\).

The high precision measurements of gravity with atom interferometers make cold atoms the most suitable test masses. Besides that, the gravity induced quantum effect is the best combination of gravity and quantum mechanics. In 1999, Chu’s group compared the values of the gravitational
acceleration measured by cold atoms with that measured by an absolute gravimeter FG-5.\textsuperscript{51} The two kinds of test masses showed the same acceleration at the level of $10^{-9}$. That comparison is called as modern version of Galileo’s “Leaning Tower” experiment. Demonstration of WEP test with two kinds of atoms was in 2004.\textsuperscript{54} $^{85}$Rb and $^{87}$Rb atoms were used as test masses. Not only the accelerations of the two kinds of atoms were measured, but also the accelerations of $^{85}$Rb at different hyperfine ground states were measured. They both proved the WEP is founded at the level of $10^{-7}$. Although the precision of this test is much lower than the test with macroscopic bodies, it showed that both particles with different internal nuclear structure and particles with different orientation of nuclear to electron spin, obey the law of free fall. Another terrestrial high precision atomic interferometer-based WEP test experiment has been prepared at Stanford University. They will build a 10 meter atom interferometer, the differential acceleration of free falling $^{85}$Rb and $^{87}$Rb atoms will be measured to test WEP,\textsuperscript{184} the expected precision is $10^{-15}$. A different 10 m interferometer was designed and developed in China,\textsuperscript{185} the test masses could either be $^{85}$Rb and $^{87}$Rb, or be rubidium and lithium atoms with more different compositions and qualities. An airborne interferometer testing WEP was designed in 2009,\textsuperscript{186} they planned to use potassium and rubidium atoms as the test masses, and put the whole device in a plane to produce a 0g environment. Recently they had successfully achieved gravity measurement on the aircraft.\textsuperscript{187}

Space offers unique experimental conditions to explore the foundations of modern physics with accuracy far beyond that of ground-based experiments, such as better seismic conditions, micro-gravity environment. Some theories considered that, the violation of EP may be more obvious in space than on the ground. Six space projects on precision WEP test have been proposed. In accordance with the expected precision from low to high, they are MicroSCOPE (Micro-Satellite à tranînée pour l’Obeservation du Principle d’Equivalence) with targeted precision of $10^{-15}$,\textsuperscript{188} POEM (Principle of Equivalence Measurement) with targeted precision of $10^{-16}$,\textsuperscript{189} MWXG (Matter wave explorer of gravity) with targeted precision of $10^{-16}$,\textsuperscript{190} STUFF (Space Test of Universality of Free Fall) with targeted precision of $10^{-17}$,\textsuperscript{191} GG (Galileo Galilei) with targeted precision of $10^{-17}$,\textsuperscript{192} STEP (Satellite Test of the Equivalence Principle) with targeted precision of
Among them, the MWXG project is the atom interferometric WEP test space project.

9.2. Local Lorentz invariance test

LLI is not only one of basic parts of the equivalence, but also the basic hypothesis of special relativity. The nonvanishing parameter $\delta = |c^2/c_i^2 - 1|$ is used to test the violation of LLI, where $c$ is the speed of light and $c_i$ is the maximum speed of the particles. And the nonvanishing parameterized post-Newtonian parameter $\alpha_3$ can also be used to test the violation of LLI. So the LLI can be test by method in high-energy field or by method in cosmophysics. There are still many other parameters in different model to test LLI.

According to the description of LLI, consequent comparison of the rates of two clocks will test LLI. Atom clock can achieve a very high precision, LLI test experiment by atom clocks was demonstrated in 1997\cite{194}, a cesium clock and a rubidium clock carried on global positioning system (GPS) satellites were compared with a hydrogen maser, the upper limit value of $\delta c/c$ is not more than $10^{-9}$, where $\delta c$ is the difference between the observed speed of light that propagates along a particular direction with the clock, and $c$ is the speed of light in vacuum. ACES takes the LLI test as one of its tasks\cite{140}, a rubidium clock and a hydrogen maser will be launched to the ISS in 2013 or 2014. Due to the development of the femtosecond laser frequency combs, optical clocks came into the sight of precision measurement of time and frequency. Not only the laser cooled neutral atoms, but also the trapped ions are used in optical clocks. This new generation clocks show higher accuracy and stability, and are better candidates for space missions.

Besides the comparison of clocks, the test of LLI with atom interferometers had been proposed\cite{195,196}, and the expected accuracy will be higher due to their high sensitivity in gravity measurement.

9.3. Test of local position invariance

The description of LPI states that the rate of a free falling clock will be the same with the rate of a standard one. In other words, comparison of two clocks with different acceleration can also test the LPI. The precision of LPI test is at the same level as the measurement of the gravitational redshift,
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it will not be better than the precisions of UFF and LLI test. In 1980, the comparison between two hydrogen clocks was performed to test LPI, \(^{197}\) one clock was carried on a suborbital rocket, and the other one was on the ground. The relative frequency shift of two clocks, \(\Delta v/v\), was disagreed with the prediction value at the level of \(10^{-5}\), where \(\Delta v\) is the gravitational redshift, \(v\) is the frequency shift due to free fall. LPI test is also one task of ACES, and the expected test precision is \(10^{-6}\).

In 2010, Müller claimed that the atom interferometer can be used to measure gravitational redshift, \(^{198}\) furthermore this measurement would achieve an accuracy of \(10^{-9}\), which is much higher than current accuracy measured with atom clocks. And this announcement caused a lively discussion.\(^{199–201}\)

9.4. Other tests

The atom interferometry is a powerful tool to sensor the phase change, any phase shifts induced by interactions between the atoms and the environment can be measured by atom interferometers. Recently, the phase shift induced by the atom-surface van der Waals interaction was measured with atom interferometers,\(^{202}\) the berry phase and Aharonov–Anandan phase,\(^ {203}\) polarizability of Na, K and Rb\(^ {204}\) were also measured with atom interferometers. Due to high precision of gravity measurement, atom interferometers will be a good tool to test the violation of the Newton’s inverse square law in mid-distance range.\(^ {205}\) The measurements of variation of \(\alpha\) with atom clocks may be used to infer the violations of fundamental symmetries.\(^ {206}\) Other potential applications of cold atoms are too numerous to mention individually.

10. Summary

Benefits from the technology of laser cooling and trapping of neutral atoms, cold atoms provide more opportunities in precision measurements. Atom transitions can provide the best frequency standards. Atoms clocks in space will perform unprecedented improvements and have an irreplaceable contribution to the GPS. Cold atom microwave clocks had achieved remarkable performance, while optical clocks have the best performances either in stability or in accuracy, and have great potential for further improvement. The rest mass that the matters have but the photons do not have
lead a great difference between matter waves and light waves, so that atom interferometers will be widely used in gravity measurements. The quantum effects induced by gravity will provide a good platform for merging the gravity and quantum mechanics together. The main factors that limit the sensitivity of an atom interferometer are the uncertainties of the lasers which are used to manipulate the atoms, and the vibration of the mirrors which is used to reflect the laser beams. The parasitic accelerations, such as the inertial acceleration and relative acceleration of the reference, are the main factors that limit the accuracy of an atom interferometer. To promote the precision of measurements, some space atom interferometer projects are proposed. Miniaturization is an important development trend of atom interferometers. Cold atoms and BECs will be used in fundamental physical constants measurements, fundamental physics principle test and other precision measurements.

References
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