Slow dark optical solitons caused by cross talk among optical transitions

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Received 28 February 2005; received in revised form 26 April 2005; accepted 14 June 2005

Abstract

We study the formation of slow dark optical solitons in a two-photon resonance Raman scheme. For a four-level atomic medium with two ground and two closely separated excited states, two transitions between one ground state and two excited states are coupled simultaneously by a strong cw laser field. This cross talk among optical transitions can modify the linear and nonlinear responses of another weak pulse probing the transition between the other ground state and one excited state. Under two-photon resonance condition and with appropriate one-photon detuning, we can obtain cancellation of the linear absorption, enhancement of Kerr nonlinearity, and slow group velocity propagation of the weak probe pulse. As a consequence, slow dark optical solitons can be formed.

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PACS: 42.50.Gy; 42.65.–k; 42.81.Dp

Keywords: Cross talk; Slow group velocity; Kerr nonlinearity; Dark optical solitons

1. Introduction

Dispersive and absorptive properties of an atomic medium can be dramatically modified by coherence and interference induced by applying external fields. In recent years, there has been a great deal of interest in the propagation of light pulses through the modified media. Using the technique of electromagnetically induced transparency (EIT) [1] or Raman-assisted interference effects, the group velocity of a light pulse can be largely reduced and even stopped [2–4]. Associated with cancellation of linear absorption and reduced
group velocity, many studies have shown that optical nonlinearities can be largely enhanced at low light intensities [5], such as cross-phase modulation (XPM) [6], enhancement of self-modulation Kerr effect [7], and multi-wave mixing [8–10]. In addition, combining slow light propagation and enhanced nonlinear effects, Wu and Deng [11,12] have theoretically proposed that it is possible to form ultraslow optical bright and dark solitons for weak light in Raman scheme.

As is well known, if the nonlinear refractive index can exactly compensate group velocity dispersion (GVD), it will result in optical solitons which keep pulse shape preserving propagation [13]. The third-order nonlinear self-phase modulation effect is the key to the formation of solitons. As used in [11,12], there are, in general, two ways [14] to enhance Kerr nonlinearities (including cross-Kerr effect and self-modulation Kerr effect). One way is applying another laser field [6] and the other way is taking a slight two-photon detuning [7,15], to disturb the two-photon resonance condition in a three-level A EIT case or Raman scheme.

In this paper, we propose a new approach to form slow optical solitons. The same as [11,12], our system is also based on two-photon resonance Raman scheme, which can cancel linear absorption and reduce group velocity of a weak probe pulse. However, the self-phase modulation nonlinearity is different from the two general ways and comes from the cross talk among optical transitions. In most studies, one laser just acts on one transition. In practice, more complicated situations exist for real atomic levels. Multi-coupling of one laser due to the nearby hyperfine levels has been studied in EIT [16]. Some other studies have shown that this common coupling can cause gain [17], optical bistability [18], and eliminating nonlinear phase shift [19]. Here, for a two-photon resonance Raman configuration composed by a strong cw laser field and a weak pulse light in a four-level atomic medium, the strong cw laser field can couple two transitions. The double coupling of the strong cw laser field largely enhances the third-order Kerr nonlinearity of the weak probe pulse. Through the nonlinear Schrödinger equation, we demonstrate the formation of dark optical solitons propagating with very slow group velocity.

2. Model and equations

We consider a medium with two ground states |1⟩ and |2⟩, and two excited states |3⟩ and |4⟩, shown schematically in Fig. 1. A strong cw laser field $E_c$ can drive two transitions |2⟩ ↔ |4⟩ and |2⟩ ↔ |3⟩ simultaneously. State |1⟩ is coupled to state |4⟩ by a weak probe pulse $E_p$. The transition |1⟩ ↔ |3⟩ is dipole forbidden transition due to selection rules. In the dipole and rotating-wave approximations, the Hamiltonian of the system in the interaction representation is given by [20]

$$H = -\Delta_p |4⟩⟨4| - (\Delta_p + \delta) |3⟩⟨3| - (\Delta_p - \Delta_c) |2⟩⟨2| - (\Omega_{c1} |4⟩⟨2| + \Omega_{c2} |3⟩⟨2| + \Omega_p |4⟩⟨1| + H.c.).$$

(1)

where $\delta$ is the frequency separation of two excited states, $\Delta_c = \omega_c - \omega_{42}$ and $\Delta_p = \omega_p - \omega_{41}$ are single-photon detunings of driving and probe field with transitions |2⟩ ↔ |4⟩ and |1⟩ ↔ |4⟩, respectively. $\Omega_{c1} = \mu_{42} E_c / 2\hbar$, $\Omega_{c2} = \mu_{32} E_c / 2\hbar$, and $\Omega_p = \mu_{41} E_p / 2\hbar$ are one-half Rabi frequencies for the respective transitions, with $\mu_{mn}$ ($m,n = 1, 2, 3, 4$) denoting the dipole moment for the corresponding transition |m⟩ ↔ |n⟩. From the Schrödinger equation, we get a set of atomic equations of motion.

Fig. 1. Energy schematic diagram of a four-level system with two ground states and two closely separated excited states. There are three dipole allowed transitions |2⟩ ↔ |4⟩, |2⟩ ↔ |3⟩, and |1⟩ ↔ |4⟩ between ground and excited states. When the Rabi frequency of the strong cw laser field $E_c$ is comparable with the separation $\delta$ of the two excited states, $E_c$ can act on two transitions |2⟩ ↔ |4⟩ and |2⟩ ↔ |3⟩ simultaneously. A weak pulse probes |1⟩ ↔ |4⟩ transition.
\[
\begin{align*}
\frac{\partial A_1}{\partial t} &= i\Omega_p A_4, \\
\frac{\partial A_2}{\partial t} &= -[\gamma_2 - i(A_p - A_c)]A_2 + i\Omega_{c1}^* A_4 + i\Omega_{c2} A_3, \\
\frac{\partial A_3}{\partial t} &= -[\gamma_3 - i(A_p + \delta)]A_3 + i\Omega_{c2} A_2, \\
\frac{\partial A_4}{\partial t} &= -(\gamma_4 - iA_p)A_4 + i\Omega_p A_1 + i\Omega_{c1} A_2.
\end{align*}
\] (2a) (2b) (2c) (2d)

Here, \( A_j \) is the \( j \)th atomic wavefunction amplitude. \( \gamma_2 \) is the decay rate of the \([2] - [1]\) coherence. \( 2\gamma_2 \) and \( 2\gamma_3 \) are the decay rates of excited states \([4]\) and \([3]\), respectively.

In the following calculations, it is assumed to be sufficiently weak that the ratio \((|\Omega_p|/|\Omega_{c1}|)^2 \) is much less than unity (weak probe approximation), so that in essence all of the atomic population remains in the ground state \([1]\). Also, the excited state \([4]\) can be adiabatically eliminated when the variation of the probe field’s envelope is slow compared to the excited state lifetime, so there is no population transfer of the ground state \([1]\) to the excited state \([4]\). Thus, we can show that \( A_4^{(0)} \approx 1, A_2^{(0)} A_3^{(0)} = 0 \) (to the zero order of \( \Omega_p \)), and \( A_1^{(1)} = 0 \) (to the first order of \( \Omega_p \)). We keep two-photon resonance \( (A_p = A_c = A) \) and get the solutions of \( A_j \) to the first order of \( \Omega_p \) from Eqs. (2b)–(2d).

\[
\begin{align*}
A_2^{(1)} &= -\frac{\Gamma_3 \Omega_{c1} \Omega_p}{i\Gamma_2 \Gamma_3 \Gamma_4 + \Gamma_3 |\Omega_{c1}|^2 + \Gamma_4 |\Omega_{c2}|^2}, \\
A_3^{(1)} &= \frac{i\Omega_{c2} \Omega_p}{i\Gamma_2 \Gamma_3 \Gamma_4 + \Gamma_3 |\Omega_{c1}|^2 + \Gamma_4 |\Omega_{c2}|^2}, \\
A_4^{(1)} &= -\frac{i(\Gamma_1 \Gamma_2 \Gamma_3 + |\Omega_{c2}|^2 \Omega_p)}{i\Gamma_2 \Gamma_3 \Gamma_4 + \Gamma_3 |\Omega_{c1}|^2 + \Gamma_4 |\Omega_{c2}|^2},
\end{align*}
\] (3a) (3b) (3c)

where \( \Gamma_2 = -\gamma_2, \Gamma_3 = -[\gamma_3 - i(A + \delta)] \) and \( \Gamma_4 = -(\gamma_4 - iA) \).

3. Linear and third-order nonlinear susceptibilities

The induced polarization at the probe frequency is \( P(\omega_p) = \chi(\omega_p)E_p \). The susceptibility is written as
\[
\chi(\omega_p) = \chi^{(1)}(\omega_p) + 3\chi^{(3)}(\omega_p)|E_p|^2,
\] (4)

where we just consider the susceptibility to the third-order and neglect the higher orders. The first-order susceptibility \( \chi^{(1)}(\omega_p) \) of the probe pulse is expressed as
\[
\chi^{(1)}(\omega_p) = K(\Omega_1^{(1)} A_1^{(1)*}/\Omega_p) = -\frac{i\Gamma_2 \Gamma_3 + \Omega_{c2}^2}{\Gamma_2 \Gamma_3 \Gamma_4 + \Gamma_3 |\Omega_{c1}|^2 + \Gamma_4 |\Omega_{c2}|^2},
\] (5)

where \( K = N|\mu_{42}|^2/h \) and \( N \) is the atomic density. For cold atoms, the decay rate of the \([2] - [1]\) coherence is very weak \( (\gamma_2 \ll \gamma_5, \gamma_4) \). In order to get a simple result of Eq. (5), we can neglect the decoherence term by taking \( \Gamma_2 = 0 \) and get
\[
\chi^{(1)}(\omega_p) \approx -\frac{i\Omega_{c2}^2}{\Gamma_3 |\Omega_{c1}|^2 + \Gamma_4 |\Omega_{c2}|^2}.
\] (6)

We can see that the first-order susceptibility \( \chi^{(1)}(\omega_p) \) is mainly caused by double coupling of the driving field. When single-photon detuning \( \Delta = 0 \), Eq. (6) in expression is similar to the cross-Kerr nonlinearity in “\( \pi \)” configuration \([6,20]\), \( \Omega_{c2} \) corresponds to the signal field, and \( \Omega_{c1} \) corresponds to the coupling field. Defining \( \eta = |\mu_{42}|^2/|\mu_{32}|^2 \), Eq. (6) reduces to
\[
\begin{align*}
\text{Re}[\chi^{(1)}(\omega_p)] &= -\frac{\delta\eta + (1 + \eta)\Delta}{(\eta\gamma_3 + \gamma_4)^2 + [\delta\eta + (1 + \eta)\Delta]^2}, \\
\text{Im}[\chi^{(1)}(\omega_p)] &= \frac{\eta\gamma_3 + \gamma_4}{(\eta\gamma_3 + \gamma_4)^2 + [\delta\eta + (1 + \eta)\Delta]^2}.
\end{align*}
\] (7a) (7b)

It shows that the absorption \( (\propto \text{Im}[\chi^{(1)}(\omega_p)]) \) and dispersion \( (\propto \text{Re}[\chi^{(1)}(\omega_p)]) \) of the probe field do not depend on the intensity of the driving field, \( \chi^{(1)}(\omega_p) \) is a linear susceptibility. When \( \Delta = -\delta\eta/(1 + \eta) \), there appears a absorption peak \( \text{Im}[\chi^{(1)}(\omega_p)]_{\text{max}} = K(\eta\gamma_3 + \gamma_4)/(\eta\gamma_3 + \gamma_4)^2 \) and the corresponding dispersion is zero. From Eq. (5), \( \text{Im}[\chi^{(1)}(\omega_p)/K] \) and \( \text{Re}[\chi^{(1)}(\omega_p)/K] \) versus the single-photon detuning \( \Delta \) are plotted in Fig. 2. Far away from the point \( \Delta = -\delta\eta/(1 + \eta) \), the linear absorption will be near zero. These features behave similar to the absorption and dispersion of one laser field interacting with a two-level system.
Following the methods used in [11,12], the third-order nonlinear susceptibility can be written as

$$\chi^{(3)}(\omega_p) = -\frac{1}{3} K' A_4^{(3)} \left( |A_4^{(1)}|^2 + |A_3^{(1)}|^2 + |A_2^{(1)}|^2 \right) \frac{1}{|\Omega_p|^2 \Omega_p}.$$  

(8)

where $K' = N|\mu_{44}|^2/\hbar^3$. For the simple three-level $A$ configuration $|1\rangle$, $|2\rangle$ and $|4\rangle$, $\Omega_{c1}$ and $\Omega_p$, the third-order nonlinear susceptibility is zero under the two-photon resonance condition [7]. But in this four-level two-photon resonance Raman system, the coupling of the driving field with transition $|2\rangle \leftrightarrow |3\rangle$ ($\Omega_{c2}$) destroys the coherence between state $|1\rangle$ and $|2\rangle$ induced by $\Omega_{c1}$, which causes the linear absorption of the probe field and also leads to the nonlinear effect.

We perform a numerical calculation of the third-order nonlinear susceptibility. Fig. 3(a) shows the curves of $\text{Im}[\chi^{(3)}(\omega_p)/K']$ and $\text{Re}[\chi^{(3)}(\omega_p)/K']$ versus the single-photon detuning $\Delta$. As a comparison, the third-order nonlinear susceptibility of a regular two-level atomic medium is plotted in Fig. 3(b). When $\Delta = 30\gamma$, we have $\text{Re}[\chi^{(3)}(\omega_p)/K'] = 4.9 \times 10^{-3}$ for the four-level system, but $\text{Re}[\chi^{(3)}(\omega_p)/K'] = 3.7 \times 10^{-5}$ for a regular two-level atomic medium. We can see that the nonlinear Kerr effect has been largely enhanced in this two-photon resonance Raman scheme.

4. Slow dark optical solitons

If the losses of the probe pulse are small enough and can be neglected, the balance between the nonlinear self-phase modulation and group velocity dispersion eventually cause the probe pulse keep shape preserving propagation to form a soliton. From above discussions, as long as single-photon detuning $\Delta$ is far away from the point $-\delta n/ (1+\eta)$, the linear absorption ($\propto \text{Im}[\chi^{(1)}(\omega_p)]$) of the probe pulse is negligible and the nonlinear self-phase modulation is enhanced. So the system is possible to form solitons for a weak probe pulse.

In the slowly varying amplitude approximation $(\partial E_p/\partial t \ll \omega_p E_p)$, $(\partial E_p/\partial z \ll k|E_p|)$, the wave equation of the slowly varying envelope $E_p(z, t)$ of the probe pulse along the $z$-axis is given by [21]

$$\frac{\partial}{\partial z} \left( \frac{1}{v_g} \frac{\partial}{\partial t} \right) E_p + \frac{i}{2} \beta_2 \frac{\partial^2}{\partial t^2} E_p = i \frac{2 \omega_p}{c} n_2 |E_p|^2 E_p,$$

(9a)

$$\frac{1}{v_g} \frac{\partial}{\partial t} \left( \text{Re} \left[ \frac{dk}{d\omega} \right] \right) = \frac{\text{Re}[n_0 + \omega_p \text{dn}_0/d\omega]}{c},$$

(9b)

$$n_0(\omega_p) = \sqrt{1 + 4\pi \chi^{(1)}(\omega_p)},$$

(9c)

$$\beta_2 = \frac{d^2 k}{d\omega^2},$$

(9d)

$$n_2 = \frac{3\pi \chi^{(3)}(\omega_p)}{n_0(\omega_p)},$$

(9e)

where $k = \omega_p n_0(\omega_p)/c$ is the wave vector, $n_0(\omega_p)$ is the linear index of refraction, $c$ is the light velocity in vacuum, $v_g$ is group velocity of the probe pulse, $\beta_2$ is the group velocity dispersion, and $n_2$ is Kerr non-linear refractive index. Group velocity $v_g$ and group velocity dispersion $\beta_2$ are mainly determined by $\text{Re}[\chi^{(1)}(\omega_p)/d\omega_p]$ and $\text{d}^2 \chi^{(1)}(\omega_p)/d\omega_p^2$, respectively. Figs. 4(a) and (b) show the curves of $|d\chi^{(1)}(\omega_p)/d\omega_p|/K$ and $|d^2 \chi^{(1)}(\omega_p)/d\omega_p^2|/K$ versus the single-photon detuning $\Delta$. As can be seen from these graphs, when far away from the point $-\delta n/ (1+\eta)$, $\text{Re}[d\chi(\omega_p)/d\omega_p] > \text{Im}[d\chi(\omega_p)/d\omega_p] = 0$ which implies slow group velocity propagation of the probe pulse, and $|\text{Re}[d^2 \chi(\omega_p)/d\omega_p^2]| > \text{Im}[d^2 \chi(\omega_p)/d\omega_p^2] \approx 0$.

We get the transformation of Eq. (9a) by defining $\xi = z$ and $\tau = t - z/v_g$.
From Figs. 3(a) and 4(b), the coefficients $b_2$ and $W$ of Eq. (10a) can be considered as real when $\Delta$ is far away from the point $-\delta \eta/(1 + \eta)$. Under this condition, Eq. (10a) is a standard nonlinear Schrödinger equation and has solitary-wave solutions. The fundamental bright soliton solution is

$$\frac{\partial \Omega_p}{\partial \xi} + \frac{i}{2} b_2 \frac{\partial^2 \Omega_p}{\partial \tau^2} = i W|\Omega_p|^2 \Omega_p,$$

$$W = -2 \pi \frac{\alpha_p}{c} \frac{K}{n_0} \frac{A_2^{(1)}|A_2^{(1)}|^2 + A_3^{(1)}|^2 + A_2^{(1)}|^2}{|\Omega_p|^2 \Omega_p}.$$  \hspace{1cm} (10b)

From Figs. 3(a) and 4(b), the coefficients $b_2$ and $W$ of Eq. (10a) can be considered as real when $\Delta$ is far away from the point $-\delta \eta/(1 + \eta)$. Under this condition, Eq. (10a) is a standard nonlinear Schrödinger equation and has solitary-wave solutions. The fundamental bright soliton solution is

$$\Omega_p = \Omega_{p0} \sec h(\tau/\tau_0) \exp(i\kappa \xi),$$

$$|\Omega_{p0}\tau_0|^2 = -\frac{\beta_2 \kappa}{W}, \quad \kappa = -\frac{\beta_2 \kappa}{2\tau_0},$$  \hspace{1cm} (11b)

where $\beta_2 \kappa$ and $W$ represent the real parts of $\beta_2$ and $W$, respectively. For bright solitons, it should satisfy $\beta_2 \kappa < 0$. If $\beta_2 \kappa > 0$, it leads to dark solitons. The fundamental dark soliton takes the form

$$\Omega_p = \Omega_{p0} \tan h(\tau/\tau_0) \exp(i\kappa \xi).$$  \hspace{1cm} (11c)

Figs. 3(a) and 4(b) show that $\beta_2 \kappa \times W$ is always larger than zero when $\Delta$ is far away from $-\delta \eta/(1 + \eta)$, then the system can only be used to form dark optical solitons.

As an example, we take $2\gamma_4 = 2\gamma_3 = 2\gamma = 2 \times 10^8$ s$^{-1}$, $\alpha_p K/c = 1.0 \times 10^9$ cm$^{-1}$ s$^{-1}$, which have the same order as the parameters given in [11,12], other parameters are the same as Figs. 2–4. When taking $\Delta = 30\gamma$, we get the linear absorption coefficient $\nu = 0.051$ cm$^{-1}$, group velocity $\nu_g = 1.3 \times 10^{-4}$ cm$^{-1}$ s$^{-1}$, and $|\Omega_{p0}\tau_0| = \sqrt{\beta_2 \kappa/W} \approx 55.6$. There are only two adjustable parameters (intensity of the control field and single-photon detuning $\Delta$) in this system. The other two parameters (the separation $\delta$ between the two excited states and the relative...
coupling strength \( \eta \) are fixed for a certain atomic medium. From our numerical calculations, we find that decreasing the Rabi frequency of the control field will increase the reduction of the group velocity of the probe pulse and increase the values of \( b_r, W_r \) and \( |X_{p0}|s_0| \). The control field must be strong enough to couple two transitions, on the other hand relatively lower intensity of control field can lead to better effects in formation of slow dark optical solitons. In addition, we have used assumption of \( |X_{p0}|^2 \ll |X_{c1}|^2 \) in our calculations. From Eq. (11b), there exists relationship of \( |X_{p0}|^2 = -\beta_{2r}/W_r \ll |X_{c1}|^2 \). The intensity of control field also gives a constraint of soliton’s width \( \tau_0 \). Figs. 2–4 show that the parameter \( \Delta \) has a very large range of validity. For the two fixed parameters, smaller separation \( \delta \) will be better and there is no special requirement for \( \eta \).

The cross talk of control field may be viewed as the perturbation to the two-photon resonance condition, it is the main reason in formation of slow dark optical solitons. The cross talk comes from the internal atomic factor (the closely separated two excited states). Compared with [11,12], of which the perturbations come from external factors (by introducing another laser field or taking slight two-photon detuning), the cross talk scheme is a very stable system to form slow dark optical solitons. Cold alkali metals can be used as the media. Taking \(^{85}\text{Rb}\) as an example, \(^{5}\text{S}_1/2\) \( F = 2, 3 \) are two ground states, and \(^{5}\text{P}_3/2\) \( F' = 1, 2 \) are selected as two excited states, the separation between the two excited states is 29 MHz. A strong cw laser field drives \( F = 2 – F' = 1, 2 \) transitions, and a weak pulse probes \( F = 3 – F' = 2 \) transition.

5. Conclusions

Based on the two-photon resonance Raman scheme, we have shown that quantum interference caused by cross talk of a strong cw laser field not only maintains negligible linear loss and slow group velocity but also enhances Kerr nonlinearity of another weak pulse, it can be used to form ultraslow dark optical solitons.

Acknowledgements

This work is supported by the National Natural Science Foundation of China under Grant No. 10474119, by the National Basic Research Programme of China under Grant No. 001CB309309, and also by funds from the Chinese Academy of Sciences.

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